1 (1) Set of points on the straight line

$$y = mx + c$$

Let $u = (x_1, y_1)$ and $v = (x_2, y_2)$ are two points on the straight line. So

$$y_1 = mx_1 + c$$

$$y_2 = mx_2 + c$$

Now for $0 \leq \lambda \leq 1$ ~~~~ their convex combination.

$$\lambda(x_1, y_1) + (1-\lambda)(x_2, y_2)$$

$$= (\lambda x_1 + (1-\lambda)x_2, \lambda y_1 + (1-\lambda)y_2)$$

Now

$$\lambda y_1 + (1-\lambda)y_2$$

$$= \lambda(mx_1 + c) + (1-\lambda)(mx_2 + c)$$

$$= m\{\lambda x_1 + (1-\lambda)x_2\} + c$$

Hence $\lambda u + (1-\lambda)v$ ~~~~ lie on $y = mx + c$.

Definition: We consider $\mathbb{R}^n$.

Let $(x_1, x_2, x_3, \ldots, x_n) \in \mathbb{R}^n$.

Then

$$c_1x_1 + c_2x_2 + \ldots + c_nx_n = z \Rightarrow cx = z$$

is called a hyperplane. Here ~~If $z = 0$~~ $c = (c_1, c_2, c_3, \ldots, c_n)$ and $x = [x_1, x_2, \ldots, x_n]$

If $z=0$ then $cx=0$. Then the hyperplane passes through the origin. and c is called normal to the hyperplane.

In terms of set notations we can rewrite hyperplane as. $H=\{x : cx=z\}$

<u>Th:</u> A hyperplane is a convex set.

<u>Proof:</u> We consider the hyperplane

$$H=\{x : cx=z\}.$$

Let $x_1, x_2 \in H$. Then $cx_1=z$ and $cx_2=z$.

Let $u=\lambda x_1+(1-\lambda)x_2$ be the ~~linear~~ convex combination of x_1, x_2. ~~Thus~~ $(0 \le \lambda \le 1)$

$$cu = c\{\lambda x_1+(1-\lambda)x_2\} = \lambda c x_1 + (1-\lambda) c x_2$$
$$= \lambda z + (1-\lambda) z = z.$$

Hence $u \in X$. Hence X is a convex set.

<u>Extreme point:</u> A point x of a convex set X is an <u>extreme point</u> if it cannot be expressed as a convex combination of two other points in X.

Geometrically an extreme point cannot lie on the line segment joining any other points.

If x is an extreme point then it cannot be written as $x=\lambda x_2+(1-\lambda)x_1$, $\underline{0<\lambda<1}$

<u>Adjacent points:</u> Two distinct extreme points are said to be adjacent points if they are connected by an edge.